A Two-Dimensional Non-Linear Model for Finding Thermal Diffusivity in Foods

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Introduction

Accurate characterization of thermal properties is important in food processing since a high degree of certainty in the knowledge of food temperature is necessary to insure consumer safety. A model for the analysis of an experiment involving the heating of cherry pulp in a cylindrical container was developed in Microsoft Excel with the intention of making this package available to researchers in the Food Science field for the purpose of finding the thermal diffusivity of foods. The parameter estimation scheme used here simultaneously determines thermal diffusivity and Biot number, both of which are assumed to be temperature-dependent. The thermal parameter estimates are based on transient temperature measurements from an experiment in which the cylindrical container is suddenly subjected to a pressurized steam environment. Methods used in the analysis have been taken from Beck and Arnold [1].

Experiment Description

The experiment being analyzed as part of this research involves a stainless-steel container of cherry pulp which is suddenly immersed in a pressurized steam environment. Figure 1 shows a schematic of the experimental container including the location of the two thermocouples used for recording the temperature histories. An additional temperature history of the steam environment is also recorded in the experiment and used in the analysis, since a convective boundary condition is assumed on the surface of the stainless-steel can. The overall length of the can is 73 mm and the diameter of the can is 54 mm.

The initial temperature conditions of the experiment are room temperature as the stainless-steel container is placed into a retort which subjects the experimental cylinder to the steam environment. The steam environment in the retort reaches a temperature of approximately 130^{o}C in a period of 6 minutes. The temperature on the sensor located on the surface of the can lags behind the steam temperature by as much as 34^{o}C during the initial phases of the admission of steam to the retort, but this difference drops to a fraction of a degree toward the end of the experiment, as the two temperatures nearly come to thermal equilibrium. The temperature of the center of the can, however, only rises by a few degrees from the initial conditions, remaining nearly 100^{o}C below the steam temperature at the end of the experiment.

Analysis Method

The conduction in the cylindrical container is modeled two-dimensionally using a numerical scheme, assuming no variation of temperature or heat flux circumferentially. Since the actual heat flux at any given time on the surface of the can is unknown, the thermal conductivity of the cherry pulp cannot be found. Similarly, the heat transfer coefficient and volumetric heat capacity cannot be found for this

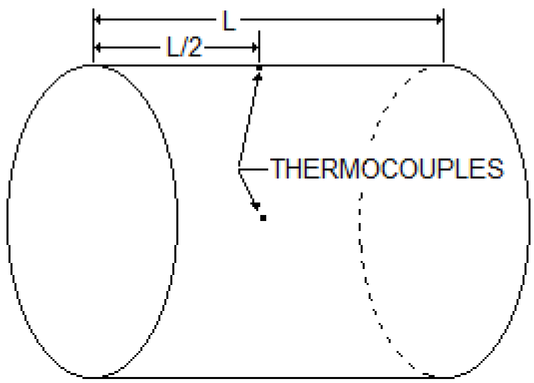

Figure 1. Experimental schematic showing the location of the temperature sensors in the stainless-steel container.

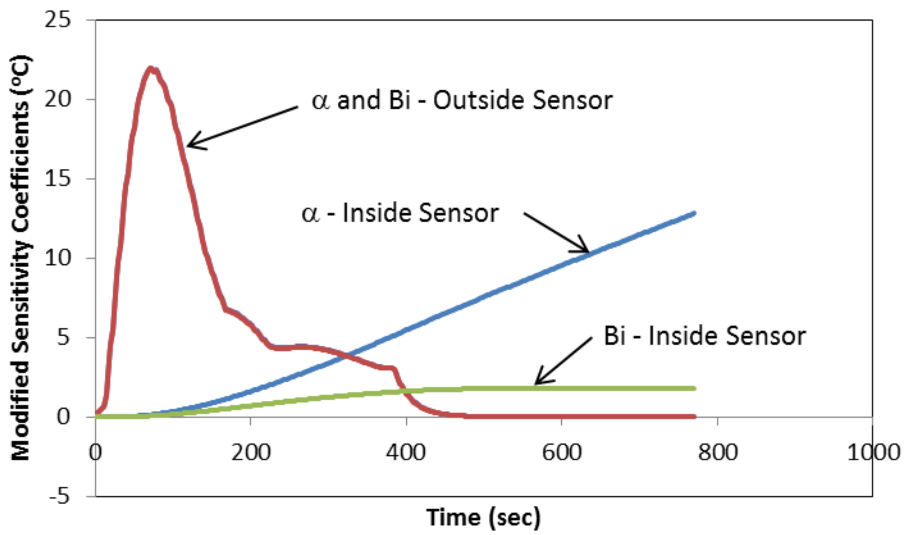

Figure 2. Sensitivity coefficients for thermal diffusivity and Biot number for each of the two sensors used in the experiment.

reason. However, the thermal diffusivity and Biot number can be found from the experimental data. Figure 2 shows the sensitivity coefficients for the parameters from the two temperature sensors used in the experiment. As can be seen in this plot, the two parameters are significantly correlated for the surface sensor but not for the internal sensor. Past analyses of similar experiments by and Dolan [2] were performed using the commercial software package COMSOL for a three-dimensional direct temperature solution which was iteratively fitted to the experimental data by trial and error. Additionally, Greiby et al. [3] utilized least-squares non-linear regression to converge a direct one-dimensional solution to the experimental data using the commercial software package PROP1D. The present research also finds converged thermal properties using least-squares non-linear regression but assumes a two-dimensional geometry. The numerical solution is programmed in Microsoft Excel and has been verified by an independent analytical solution [4]. The resulting fit between the calculated

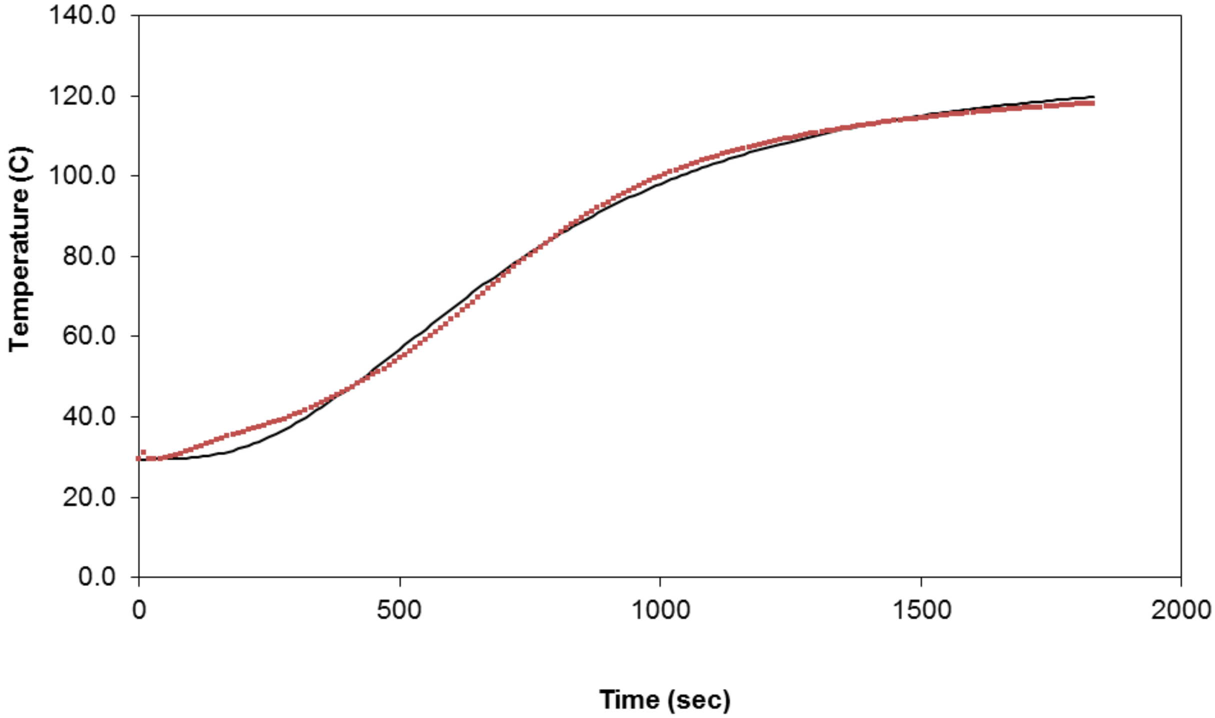

Figure 3. Measured and calculated temperatures from the experiment at the center temperature sensor location.

temperature and the recorded temperature histories is shown in Figure 3. The thermal parameters obtained from this fit were assumed to vary as linear functions of temperature. As such the value for each parameter was found at 25°C and at 100°C. Table 1 shows these results. The analysis program used for this work is intended to be made available to food science researchers involved in similar work involving the convective heating of food samples in cylindrical containers. The two-dimensional method is convenient to use and provides calculation times which are a fraction of those generated by the three-dimensional solution. It also offers the non-linear regression feature of the one-dimensional model, but accounts for the significant heat lost through the end of the enclosure, offering more accurate modeling than a one-dimensional solution. The standard deviation of the residuals using the two-dimensional method are nominally half of those generated using the three-dimensional trial-and-error fit (1.71°C vs. 2.56°C).

Table 1. Results from Parameter Estimation Analysis		
Quantity	25°C	100°C
Diffusivity (m^2/s)	2.15×10^{-7}	2.15×10^{-7}
Bi	71.5	21.4

References

[1] J. Beck and K. Arnold, *Parameter Estimation*, 1977, Wiley, New York.

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